

## FULL WAVE ANALYSIS OF MICROSTRIP FILTERS CONTAINING DIELECTRIC AND METALLIC RESONATORS WITH THE METHOD OF LINES

Arnd Kornatz, *Member, IEEE* and Reinhold Pregla, *Senior Member, IEEE*

Allgemeine und Theoretische Elektrotechnik,  
FernUniversität Hagen, D-58084 Hagen, Germany

### Abstract

In this paper it will be shown how the method of lines can be adapted to the analysis of microstrip filters. The filters consist of a dielectric lossy resonator of arbitrary shape excited by a microstrip line. Field distributions and scattering parameters will be presented.

### 1 Introduction

Although the structure of microstrip filters is very simple – a dielectric or metallic resonator is positioned beside a microstrip line – three-dimensional analysis methods are necessary in order to simulate their electromagnetic behavior. Possible methods are mode-matching methods [1], finite element methods [2], finite difference methods [3] or the method of lines [4]. An overview of most of these methods can be found in [5].

For electrodynamic problems the method of lines is a powerful tool for the analysis of multilayered structures. In every layer the permittivity may vary in two dimensions. In these two dimensions the method of lines works like a finite difference method. In the third dimension the solution is found analytically. Because of this semi-analytical property good results can be achieved with the method of lines even when only a few discretization lines are used. The interface conditions between different media are optimally fulfilled even if there are large variations in the permittivity [6] so that not only dielectric resonators can be analyzed but also lossy metallic resonators with great imaginary permittivities.

For the investigation of dielectric resonator filters it was necessary to combine several extensions of the method of lines which have never been used together or have never been used before for high frequency applications. The incoming and transmitted waves are modelled by inhomogenous [7] and special absorbing boundary conditions. The inhomogenous media make it necessary to discretize and solve a system of coupled Sturm-Liouville differential equations simultaneously. So far this has been used in a similar way only in method of lines based beam propagation algorithms [8].

### 2 Theory

Most dielectric resonators are cylinders with constant cross-sections. Therefore the microstrip filters can be described by a model of three or more layers. The first layer is the space above the resonator. The second layer is inhomogenous and contains the resonator and the remaining layers represent the multilayered substrate. Fig. 1 shows such a structure. In every layer the electromagnetic field can be derived from a vector potential  $\mathbf{A}$ . It is important that the potential

WE  
3D

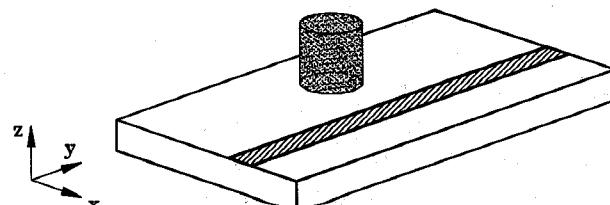


Figure 1: Principle design of a dielectric resonator filter

has the same vector components as the gradient of the permittivity of the material  $\epsilon_r$ .

$$\mathbf{A} = A_x \cdot \mathbf{e}_x + A_y \cdot \mathbf{e}_y \quad (1)$$

Only this general solution leads to a consistent system of coupled differential equations for the potential components  $A_x$  and  $A_y$ . A similar solution was presented by [9] for one dimensional permittivity variations. The extension for two dimensional permittivity variations was first used for the investigation of integrated optical structures [8]. The layers of the structure are discretized in the  $x$  and the  $y$ -direction. Therefore the discretization of the system of differential equations leads to a set of coupled ordinary differential equations

$$\frac{\partial^2}{\partial z^2} \hat{\mathbf{A}} - \hat{\mathbf{Q}} \hat{\mathbf{A}} = \hat{\mathbf{B}} \hat{\mathbf{A}}_0 \quad (2)$$

The vector  $\hat{\mathbf{A}}$  contains the discretized potential components  $A_x$  and  $A_y$  on the discretization lines. The exitation vector  $\hat{\mathbf{A}}_0$  contains the potential at the input of the filter. At the output of the filter an outgoing fundamental mode is assumed which leads to special absorbing boundary conditions that are directly used in the difference operators. Transforming eqn. (2) to main axis gives a system of uncoupled differential equations

$$\frac{\partial^2}{\partial z^2} \hat{\mathbf{A}} - \hat{k}_z^2 \hat{\mathbf{A}} = \hat{\mathbf{B}} \hat{\mathbf{A}}_0 \quad (3)$$

with

$$\hat{k}_z^2 = \hat{\mathbf{T}}^{-1} \hat{\mathbf{Q}} \hat{\mathbf{T}}; \quad \hat{\mathbf{A}} = \hat{\mathbf{T}}^{-1} \hat{\mathbf{A}} \quad (4)$$

From the solution of eqn. (3) a relation between the potential and its derivatives with respect to  $z$  at the top and the bottom of a layer can be derived. The linear equation system

$$\hat{\mathbf{Y}} \hat{\mathbf{E}}_t + \hat{\mathbf{Y}}_0 \hat{\mathbf{E}}_{0t} = -Z_0 \hat{\mathbf{J}}_F \quad (5)$$

is obtained by transforming the potential back to the spacial domain and matching the fields at the interfaces between the layers.  $\hat{\mathbf{E}}_t$  contains the discretized tangential electric field,  $\hat{\mathbf{E}}_{0t}$  the exciting tangential electric field and  $\hat{\mathbf{J}}_F$  the discretized surface current at the interfaces. From

eqn. (5) the electric field beside the metallization and the surface current on the metallization can be computed.

It is also possible to investigate isolated resonators with the presented algorithm. The computational window has to be surrounded with absorbing boundaries [10]. Eqn. (5) changes to

$$\hat{\mathbf{Y}}(f) \hat{\mathbf{E}}_t = 0 \quad (6)$$

The system matrix  $\hat{\mathbf{Y}}$  is a function of the frequency  $f$ . The system equation has non-trivial solutions if  $\det(\hat{\mathbf{Y}})$  disappears. Therefore the resonance frequencies are the zeroes of  $\det(\hat{\mathbf{Y}})$ .

### 3 Results

A metallic resonator with finite thickness which is excited by a microstrip line was chosen as an example for a dielectric resonator filter. Its finite conductance is modelled by a large imaginary permittivity. Fig. 2 shows the analyzed structure. Very few discretization lines were used for the analysis. Fig. 3 and fig. 4 show the computed

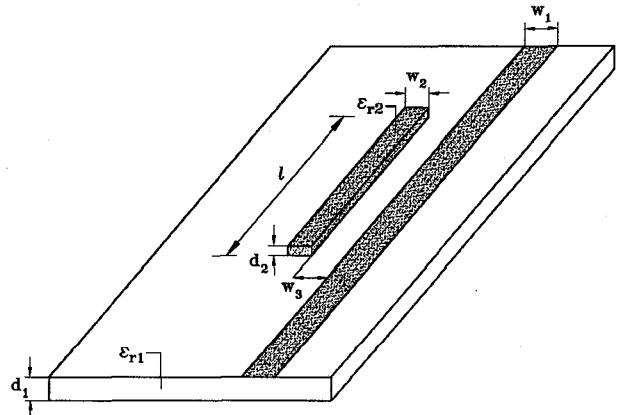


Figure 2: Lossy metallic resonator excited by a microstrip line,  $d_1 = 150\mu\text{m}$ ,  $d_2 = 30\mu\text{m}$ ,  $w_1 = 150\mu\text{m}$ ,  $w_2 = 128.6\mu\text{m}$ ,  $w_3 = 75\mu\text{m}$ ,  $l = 2.55\text{mm}$ ,  $\epsilon_{r1} = 12.9$ ,  $\epsilon_{r2} = -j 1/\rho_{Cu}\omega\epsilon_0$

run of the transmission coefficient of the filter. At the resonance frequencies the absolute value of  $S_{21}$  decreases and the phase changes abruptly. In order to verify the applied method an isolated dielectric resonator with circular shape was analyzed which is well known from other publica-

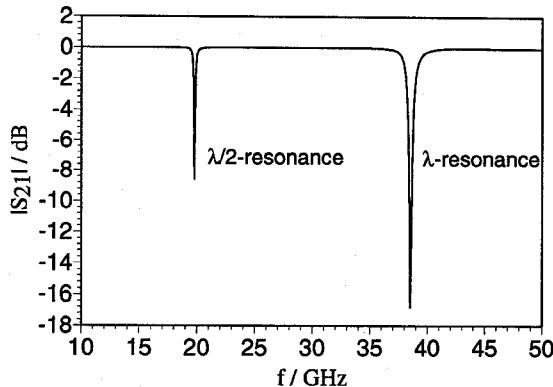


Figure 3:  $|S_{21}|$  of the lossy metallic resonator filter in fig. 2 with  $15 \times 33$  discretization lines for one component of the potential

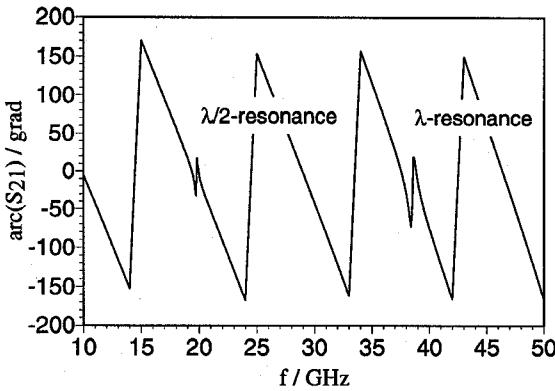


Figure 4: Phase of  $S_{21}$  of the lossy metallic resonator filter in fig. 2 with  $15 \times 33$  discretization lines for one component of the potential

	$k_0 a$	Q-factor
our Method	0.537	40.9
Theory [11]	0.537	43.73
Theory [12]	0.531	45.8
Theory [13]	0.534	40.8
Theory [14]	0.535	47
Measured [15]	0.533	46.4

Table 1: Comparison of resonance frequency and quality factor due to radiation of the  $TE_{01\delta}$ -mode of a cylindrical resonator with the results of other authors.  $\epsilon_r = 38$ , height=6.4 mm, diameter  $a=10.5$  mm

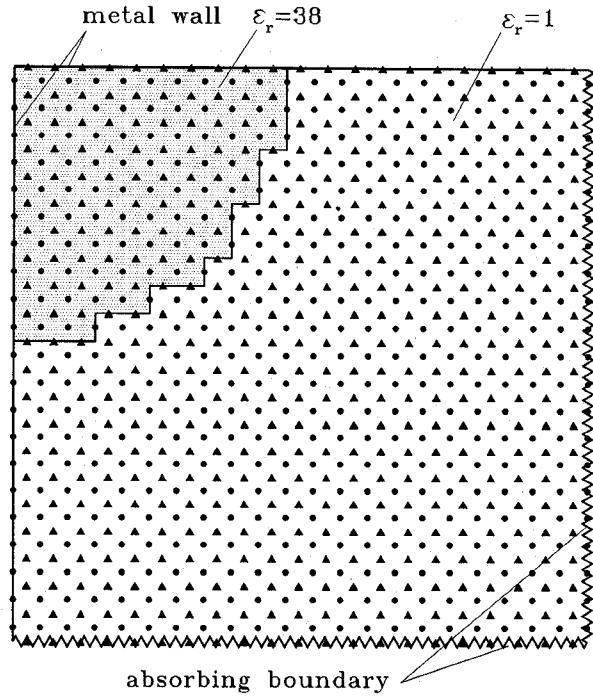


Figure 5: Discretization of the resonator with cylindrical cross section

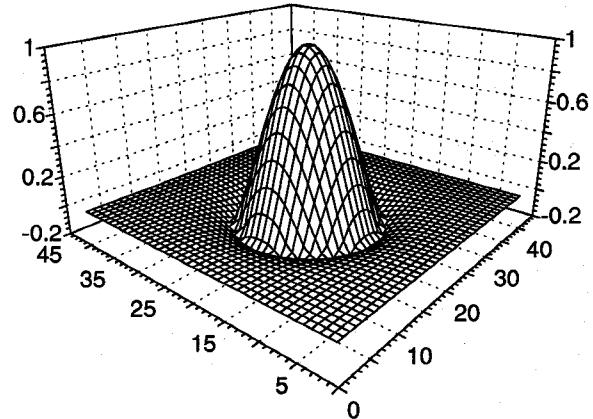


Figure 6: Real part of the normal magnetic field in the middle of the dielectric resonator

tions [11] [12] [13] [14] [15]. The approximation of the circular shape in the computation window is shown in fig. 5. We have calculated the resonance frequency and the quality factor due to radiation of the  $TE_{01\delta}$ -mode of this resonator. The normal magnetic field in the middle of the resonator is shown in fig. 6. In table 1 our results are compared with those of other authors. They

are in good consistency.

## 4 Conclusion

We have shown how to analyze microstrip lines with dielectric discontinuities by means of the full hybrid method of lines. A first result is presented. Results for isolated resonators are in good consistency with other publications. In the future we will compute more accurate scattering parameters for dielectric resonator filters in order to compare our method with other methods.

## References

- [1] U. Crombach, R. Michelfeit, "Resonanzfrequenzen und Feldstärken in geschirmten dielektrischen Scheiben- und Ringresonatoren", *Frequenz*, vol. 35, no. 12, pp. 324-328, 1981
- [2] M.V.K. Chari, P.P. Silvester (Eds.), *Finite Elements in Electrical and Magnetic Field Problems*. New York: John Wiley & Sons, 1980
- [3] J. Van Bladel, *Electromagnetic Fields*. New York: Hemisphere, 1985
- [4] R. Pregla, W. Pascher, "The Method of Lines", in T. Itoh, (editor), "Numerical Techniques for Microwave and Millimeter Wave Passive Structures", pp. 381-446, J. Wiley Publ., New York, 1989
- [5] D. Kajfez, P. Guillon, *Dielectric Resonators*, Artech House Inc. Norwood, MA, 1986
- [6] A. Kornatz, R. Pregla, "Increase of the order of approximation and improvement of the interface conditions for the method of lines". *IEEE Journal of Lightwave Technology*, vol. 11, no. 2, pp. 249-251, February 1993
- [7] S. B. Worm, "Full-wave analysis of discontinuities in planar waveguides by the method of lines using a source approach", *IEEE Transactions on Microwave Theory and Techniques*, vol. 38, pp. 1510-1514, 1990
- [8] R. Pregla, J. Gerdts, E. Ahlers, S. Helfert, "MoL-BPM algorithms for waveguide bends and vectorial fields", *Proc. Integrated Photonics Research*, New Orleans, USA, pp.32-33
- [9] R. E. Collin, "Field Theory of Guided Waves", pp. 232-244, McGraw-Hill, New York, 1960
- [10] A. Dreher, R. Pregla, "Analysis of planar waveguides with the method of lines and absorbing boundary conditions", *IEEE Microwave and Guided Wave Letters*, vol. 1, pp. 138-140, June 1991
- [11] D. Kremer, R. Pregla, "The method of lines for the hybrid analysis of multilayered dielectric resonators", *IEEE MTT-S International Microwave Symposium*, Orlando, May 1995
- [12] D. Kajfez, A. W. Glisson, J. James, "Computed modal field distributions for isolated dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1609-1616, Dec. 1984.
- [13] W. Zheng, "Computation of complex resonance frequencies of isolated composite objects," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 953-961, June 1989.
- [14] J. A. Pereda, L. A. Vielva, A. Vegas, A. Prieto, "Computation of resonant frequencies and quality factors of open dielectric resonators by a combination of the finite-difference time-domain (FDTD) and Prony's methods," *IEEE Microwave Guided Wave Lett.*, vol. 2, pp. 431-433, Nov. 1992.
- [15] R. K. Mongia, C. L. Larose, S. R. Mishra, P. Bhartia, "Accurate measurement of Q-factors of isolated dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-42, pp. 1463-1467, Aug. 1994.